

# RAPID GENERATION OF OPTIMAL ASTEROID POWERED DESCENT TRAJECTORIES VIA CONVEX OPTIMIZATION

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Mission proposals that land on asteroids are becoming popular. However, in order to have a successful mission the spacecraft must reliably and softly land at the intended landing site. The problem under investigation is how to design a fuel-optimal powered descent trajectory that can be quickly computed on-board the spacecraft, without interaction from ground control. An optimal trajectory designed immediately prior to the descent burn has many advantages. These advantages include the ability to use the actual vehicle starting state as the initial condition in the trajectory design and the ease of updating the landing target site if the original landing site is no longer viable. For long trajectories, the trajectory can be updated periodically by a redesign of the optimal trajectory based on current vehicle conditions to improve the guidance performance. One of the key drivers for being completely autonomous is the infrequent and delayed communication between ground control and the vehicle. Challenges that arise from designing an asteroid powered descent trajectory include complicated nonlinear gravity fields, small rotating bodies and low thrust vehicles.

There are two previous studies that form the background to the current investigation. The first set looked in-depth at applying convex optimization to a powered descent trajectory on Mars with promising results.<sup>6</sup> This showed that the powered descent equations of motion can be relaxed and formed into a convex optimization problem and that the optimal solution of the relaxed problem is indeed a feasible solution to the original problem. This analysis used a constant gravity field. The second area applied a successive solution process to formulate a second order cone problem that designs rendezvous and proximity operations trajectories.<sup>2</sup> These trajectories included a Newtonian gravity model. The equivalence of the solutions between the relaxed and the original problem is theoretically established.

The proposed solution is to use convex optimization, a gravity model with higher fidelity than Newtonian, and an iterative solution process to design the fuel optimal trajectory. The solution to the convex optimization problem is the thrust profile, magnitude and direction, that will yield the minimum fuel trajectory for a soft landing at the target site. The focus of this paper is how to relax and manipulate a non-convex optimization problem into a convex problem whose solution will be the solution of the original problem.

A key advantage of using convex optimization and its subclasses is that as long as there is at least one feasible solution the global minimum is guaranteed in a finite number of steps. A wide range of functions and formulations are included in the convex class and convex optimization including second order cone programming and linear programming. Common techniques for relaxing and manipulating an optimization problem into a convex form include change of variables, introduction of a slack variable, and approximation of a quantity through a Taylor series expansion.

The fuel-optimal control problem in this work is to determine the optimal (finite) thrust vector (both magnitude and direction) to land the spacecraft at a specified location, in the presence of a highly nonlinear gravity field and subject to various mission and operational constraints. The equations of motion are formulated in a rotating coordinate system and includes a higher order gravity model. The vehicle's thrust magnitude can vary between maximum and minimum bounds during the burn. Also, constraints are included to ensure that the vehicle does not run out of propellant, or go below the asteroid's surface, and any vehicle pointing

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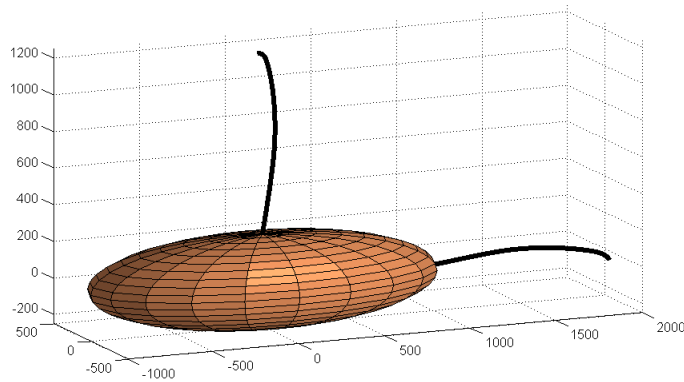
requirements. The equations of motion are discretized and propagated with Euler's method in order to produce equality constraints for the optimization problem. These equality constraints will allow the optimization algorithm to solve the entire problem, without including a propagator inside the optimization algorithm.

The gravity model was the most challenging aspect to manipulate into a convex form. The gravitational acceleration is the gradient of the gravitational potential function going up to an order of two in the summation series (2x2 model). This gradient is highly nonlinear in the position vector including radius raised to the fifth power and cross multiplication of position terms. The best way to handle these nonlinearities is to introduce a successive solution method (iterations). A series of convex optimization problems are solved with each one using data from the previous solution. The equations of motion can be rearranged as in Eq. (1), where (k) is the current iteration and (k-1) is the previous iteration.

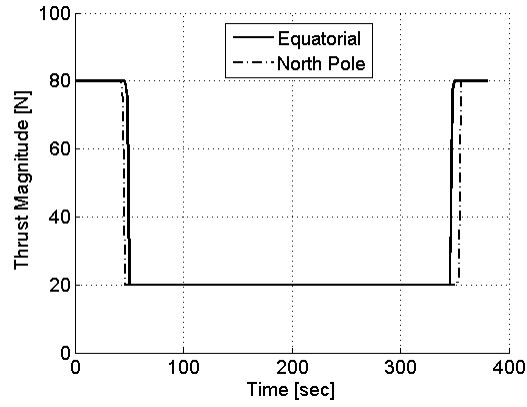
$$\dot{\vec{x}}^{(k)} = A \left( r^{(k-1)} \right) \vec{x}^{(k)} + B \vec{u}^{(k)} + c \left( r^{(k-1)} \right) \quad (1)$$

The  $\vec{x}$  is the vehicle state vector and  $\vec{u}$  is the control vector which is vehicle acceleration.  $A \left( r^{(k-1)} \right)$  corresponds to the terms that are linear in state and evaluated using the previous iteration's trajectory position vector.  $B$  contains the control vector terms which are always linear and constant.  $c \left( r^{(k-1)} \right)$  are the higher order terms in the state. The successive optimization problems are solved until the position vector from the previous iteration and the current iteration are within a pre-determined tolerance so that the two solutions are identical. At this point, the terms evaluated with the previous trajectory will be identical to having evaluated them with the current trajectory. The equations of motion are now linear equations in terms of the optimization problem. The entire problem is now in a convex form, actually in second order cone programming format. A large number of solvers and algorithms exist to solve these type of problems. Examination of the gravity model terms and where to place them in  $A$  and  $c$  was examined using five different placements. The one that worked the best and allows for extension to a higher order model is to place the Newtonian terms in  $A$  and all the remaining higher order terms in  $c$ .

Results showing trajectories designed by this method using a Matlab based convex solver will be shown to highlight the success of this method. Examples of these trajectories are located in Figure 1. This shows the position profile in 3-D for a trajectory landing at the asteroid's north pole and another landing on the equator. The corresponding thrust magnitude for these two trajectories is depicted in Figure 2.



**Figure 1. Vehicle trajectory for North Pole (top) and Equatorial (right) for a 380 second flight time. Scale is in meters.**



**Figure 2. Thrust Magnitude versus Time for a North Pole and a Equatorial trajectory.**

## REFERENCES

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